Faculty Working Papers

NUTRITION PLANNING IN DEVELOPING NATIONS:
A BICRITERION MATHEMATICAL PROGRAMMING APPROACH

H. P. Benson and T. L. Morin

#287

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

FACULTY WORKING PAPERS

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

November 12, 1975

NUTRITION PLANNING IN DEVELOPING NATIONS: A BICRITERION MATHEMATICAL PROGRAMMING APPROACH

H. P. Benson and T. L. Morin

#287

NUTRITION PLANNING IN DEVELOPING NATIONS:
A BICRITERION MATHEMATICAL PROGRAMMING APPROACH

by

H. P. BENSON* and T. L. MORIN**

October, 1975

*College of Commerce and Business Administration
The University of Illinois
Urbana, Illinois 61801

**School of Industrial Engineering
Purdue University
West Lafayette, Indiana 47907

ABSTRACT

A nutrition planning model for developing nations is presented that provides the lower income segment of the population the best possible diet in terms of caloric and reference protein intake. Furthermore, both the quality and the quantity of the protein intake are optimized. The resulting bicriterion optimization model is solved for the case of Colombia, South America, a related dual problem is analyzed, and implications for nutrition planning in Colombia are discussed.

1. INTRODUCTION

The extent of the world malnutrition problem is difficult to estimate or comprehend. Estimates suggest that perhaps one-third of the world's people are malnourished, with over 900 million suffering from deficient energy or protein intakes, or both [63, p. 28]. Individual suffering, permanent physical and mental damage, and death as a result of malnutrition are a common occurrence in many developing countries of Asia, Africa, and Latin America. Widespread malnutrition also hampers economic growth in many of these developing countries [7].

With a problem as enormous in extent and as complex in detail as malnutrition, considerable planning is required before steps can be taken to implement solutions. Recently, various quantitative techniques have been exploited for this purpose. Quantitative approaches useful for nutrition planning include models in agricultural economics ([35], [36], [65], [76], [77], [87]), least cost diet studies ([2], [10-12], [19], [20], [41], [58], [61], [69], [73]), simulation and system dynamics models ([13], [55], [56]), and mathematical programming models ([67], [68], [70], [72], [74], [75]).[1] Of these, the mathematical programming approach offers perhaps the most promise. It provides a practical approach for formulating nutrition plans that take into account the nutrient needs of the population at hand and the economic ability of the poorer, malnourished segments to purchase nutritionally-adequate diets. In addition, mathematical programming models allow for the specification of the food mix that is to be provided and of efficient methods for doing so.

This paper is organized in the following manner. §2 contains an overview of nutrient standards and a critical examination of the methods by which these nutrient standards are estimated. In §3 the implications of

these methods and their results for current mathematical programming approaches to nutrition planning are discussed. A bicriterion mathematical programming model that maximizes the value of the diet in terms of caloric and reference protein intake is presented. This approach provides the best possible diet, in terms of caloric and reference protein intake, to the population of concern. Furthermore, both the quality and the quantity of the protein intake are optimized. In §4 the model is tested utilizing data from Colombia, South America. The output of the model is a nutrition plan that provides the lower income segments of the population a considerably more beneficial diet than is presently consumed, without diminishing the nutrient intakes among upper income groups. The agricultural, economic, and trade policies and goals in Colombia receive close attention in the model. Critical resources in the nutrient supply system become apparent from the analysis, and, as a by-product of the solution technique, a number of alternate efficient nutrition plans is generated. The paper concludes with a summary and discussion in §5.

2. NUTRIENT STANDARDS: AN OVERVIEW AND CRITICAL EXAMINATION

A major consideration for nutrition planning involves the selection of nutrient supply goals. The usual approach is to calculate "required" or "recommended" levels of nutrient supplies for a population based upon recommendations of various national or international agencies. Unfortunately, the results presented by these agencies are highly tentative. Furthermore, the recommendations of any two agencies are usually not in agreement with one another.

In 1936, the League of Nations [53] became the first agency to

attempt to quantify nutritional requirement levels. By 1943, the United States began issuing periodic editions of its Recommended Dietary Allowances, the two most recent issues being the 1968 [27] and 1973 [28] editions. In 1945 the Food and Agriculture Organization (FAO) was established, and one of its tasks was to quantify, in conjunction with the World Health Organization (WHO) international standards for nutrition. Since that time a number of FAO and WHO recommendations for caloric ([22], [23], [48]) and protein ([24], [48], [49]) intakes have been issued, as well as for other nutrients ([26], [50], [51], [85], [86]). In addition, many individual governments besides the United States have issued their own nutritional standards ([9], [14], [57], [62]).[2]

The methods used to derive recommended nutritional levels vary from organization to organization, and even within the same organization at different times. This is mainly because the adequacy of the methods, and thus of the recommended results, is questionable [34].

The attempts to quantify standards for daily energy and protein intakes serve to illustrate the uncertainty of any published nutrient standards. Energy and protein allowances received the earliest attention, and many of the problems nutritionists have in establishing standards for them are also encountered when attempts are made to set recommended intakes for vitamins and minerals. Furthermore, in view of the fact that protein-caloric malnutrition (PCM) is the major malnutrition problem in the world, it is especially important that the problems involved in establishing protein and caloric standards be understood. We briefly reivew these in the following.

Calories are our source of energy.[3] Energy is required for carrying on vital body processes, including respiration, metabolism of cells, and glandular activity, for digestion, absorption, and transport of foods and

food components to the cells for metabolism, and for physical activity. Three nutrients, carbohydrates, fats, and proteins, are capable of providing energy when ingested.

The most common method for estimating adult energy needs for populations is to hypothesize the existence of a reference man and a reference woman of fixed ages and weights who live in certain environments and engage in moderate activities. From laboratory experiments that determine requirements for individuals similar to the reference man and woman, the average daily caloric requirements for the references can be calculated. Adjustments for age, activity level, and body weight can then be made.

Proteins are required for growth, maintenance, and repair of body tissues. However, if these needs have been met or fat and carbohydrate intakes are low, protein can be used as an energy source. Proteins are made of eight <u>essential amino acids</u>[4] and various <u>nonessential amino acids</u>. Both types of amino acids contain nitrogen, but essential amino acids cannot be metabolized by the human body from simpler biochemical compounds, while the nonessential amino acids can be. The nonessential amino acids supply nitrogen vital for proper bodily function, while essential amino acids act as a nitrogen source only if the essential amino acid supply is in surplus. The nitrogen supplied by nonessential amino acids and excess essential amino acids is often called <u>nonessential nitrogen</u>.[5]

The eight essential amino acids and the nitrogen contained in protein molecules are the key chemicals that the body needs. <u>Complete</u> protein foods or diets contain all eight essential amino acids in the proper proportions, with respect to total protein, that man needs. <u>Incomplete</u> proteins lack one or more of the essential amino acids in the proper amounts. If the total essential amino acid content of a daily diet is incomplete,

the efficiency with which the dietary protein is utilized is impaired. The essential amino acid content of a diet or food, then, defines the quality of its protein.[6]

There are two methods commonly used for gauging protein needs. Both involve estimating nitrogen requirements. Since protein is the major body source of nitrogen, and most protein molecules are 16 percent nitrogen, the estimated nitrogen requirement can be used to calculate an approximate protein requirement.

Each of the two methods for estimating nitrogen requirements for populations first finds the needs of a fictional reference man and reference woman. Subjects similar to the references are tested under laboratory conditions to obtain the reference requirements. These levels are then adjusted for stress, age, weight, and, in the case of children, for growth.

The factorial method for gauging nitrogen requirements requires subjects to be fed protein-free diets until the daily level of excretion of nitrogen (which occurs mostly through the elimination of urine and feces and through perspiration) stabilizes. This level is taken as the daily nitrogen requirement.

Balance studies are the second method available for estimating nitrogen needs. In these studies, subjects are fed decreasing amounts of dietary nitrogen in successive three- to seven-day periods. Each period's length is chosen so that the level of excretion of bodily nitrogen can stabilize. When stabilization occurs, a new period of lower nitrogen intake begins. The minimum nitrogen requirement is taken as that level of intake at which the stabilized level of excretion equals the amount ingested.

A critical examination of the use of these methods for estimating energy and protein requirements and the results they have given will reveal why

published recommended dietary intakes are highly tentative and vary among organizations and through time. There are a number of difficulties in using these methods.

First, different methods yield different results, and nutritionists are uncertain as to which, if any, of the methods is appropriate. For example, both the National Research Council in the United State [27] and the FAO [49] have used the factorial method to determine protein standards. However, in 1973 the FAO considered the results from both the factorial method and balance studies. The balance studies implied requirements were one-third higher than those predicted by the factorial method. They concluded that at nitrogen balance, "the efficiency of nitrogen utilization is appreciably lower than when protein intake is low," [48, p. 53] as it is in the factorial method. They tentatively chose to base their results on the balance studies, although the factorial approach had been the basis for their estimates eight years earlier.

Another problem in hypothesizing fixed requirement levels for energy and protein is that individuals can adapt to various levels of intake. For example, basal metabolic rates and physical activity decrease when caloric intake is lowered [23, p. 7], [32, p. 87], [48, p. 19]. Strong evidence exists that individuals can also adapt to decreased protein and essential amino acid intakes, perhaps by more extensively reutilizing essential amino acids released by the body during tissue breakdown and by a reduction in the rate of destruction (catabolism) of essential amino acids [66, p. 1603]. If individuals do, in fact, adapt to different levels of protein intake, then neither approach for estimating protein needs is adequate. In the factorial method, to compensate for a diet free of nitrogen, an individual would reduce nitrogen excretion, so that this method would underestimate

daily needs. Balance studies would also be inadequate, since an individual could achieve nitrogen equilibrium at any number of levels.

In addition, climatic effects upon energy and protein needs are unknown, and the effects of stress and infection have yet to be quantified [48, pp. 27, 69, 99], [49, pp. 13, 18, 28, 32]. Furthermore, the efficiency with which calories and proteins are utilized at various levels of intake and when other nutrients are marginal or deficient is unknown [32, p. 12], [34, p. 156], [38, p. 37], [49, pp. 6, 41]. The best definitions to use for the reference man and woman are unknown and continue to change as experts modify their results. Also, some levels of recommendation are not meant to imply that individuals who do not meet them are malnourished [27, p. v], while others do imply this [48, pp. 11-12]. Even the terms applied to the various published levels, including recommended, safe, minimum, average, optimum, and required levels of intake, reflect the uncertainties in the experimental and observational methods and results [27], [48, p. 10], [49, p. 11]. All of these factors explain why levels of energy and protein intake recommended by various organizations are highly tentative and are often at variance with one another.

There is one more factor unique to the specification of protein requirements that deserves special attention. In establishing protein requirements, a level of intake is set for protein that is of perfect quality in that it is fully utilized. Given this level and the protein quality of actual diets, the actual level of protein intake that ought to be ingested can be computed. This level will be higher than the level of perfect quality protein recommended in order to compensate for the fact that diets are not of perfect protein quality.

The essential amino acid balance of a diet or food is used to define its protein quality. The protein in eggs and human milk is thought to be

100 per cent utilized, so that the essential amino acid pattern of these proteins has been identified as the ideal, or reference protein [49, p.37]. However, recent evidence suggests that other factors besides the essential amino acid pattern may play a role in the efficiency of utilization of dietary protein. Studies have shown that the efficiency of utilization may increase as protein intake approaches marginal levels and decrease at high levels of intake [48, p. 53], [54, p. 577], [64, p. 1363]. Furthermore, the efficiency of utilization may depend upon the age of the individual [48, p. 71]. If these hypotheses are true, then the present definition of protein quality is not adequate. A precise knowledge of the efficiency with which ingested protein is utilized is required before meaningful levels of intake can be recommended.

In view of these uncertainties as to proper methods for evaluating nutrient requirements and the tentative nature of the recommendations that have been issued, nutrition planners must exercise a considerable degree of caution when employing nutrient standards. Indeed, when mathematical programming techniques for nutrition planning are employed, the problems are serious enough to warrant an approach that does not depend heavily upon prespecified requirement levels.

3. A BICRITERION MATHEMATICAL PROGRAMMING APPROACH

The bicriterion mathematical programming approach presented in this section provides a method for nutrition planning that yields the best possible diet, in terms of energy and reference protein intake, for the population of concern. An examination of the implications of our incomplete knowledge of nutrient requirements for typical mathematical programming approaches to nutrition planning will reveal the motivation for this bicriterion model.

3.1 Traditional Approaches

Mathematical programming approaches to nutrition planning ([67], [68], [70], [72], [74], [75]) typically involve the optimization of a single objective function subject to resource and nutritional constraints. Consider a simplified version of a national nutrition planning model. Let $x \in \mathbb{R}^n$ denote the nutritional "plan," a decision vector specifying the levels of agricultural, processing, distribution, and consumption activities. For each $j \in \{1,2,\ldots,p\} = J$, let $f_j(x): \mathbb{R}^n \to \mathbb{R}$ represent the amount of nutrient j that is available per year for human consumption under plan x, and, for each $i \in \{1,2,\ldots,m\} = I$, let $g_i(x): \mathbb{R}^n \to \mathbb{R}$ represent the annual level of consumption of resource i under plan x. Let $K = \{1,2,\ldots,n\}$. Assume that the annual cost of nutrition plan x is given by a function $c(x): X \to \mathbb{R}$, where $X = \{x \in \mathbb{R}^n \mid f_j(x) \geqq r_j \; \forall j \in J;\; g_i(x) \leqq b_i \forall i \in I;\; x_k \geqq 0 \; \forall k \in K\}$. Then, a typical mathematical programming model (P) for nutrition planning involves finding a plan x so as to

$$\min c(x)$$

subject to

$$
\begin{aligned}
f_j(x) &\geqq r_j \quad \forall j \in J \\
g_i(x) &\leqq b_i \quad \forall i \in I \qquad\qquad (P) \\
x_k &\geqq 0 \quad \forall k \in K,
\end{aligned}
$$

where r_j represents the annual requirements for nutrient $j \in J$, and b_i represents the amount of resource $i \in I$ available.[7] Although the nutrient requirement levels are highly tentative and may vary from organization to organization, nutrition planners employing models such as (P) must, nevertheless, choose among the various standards. Let us examine the weaknesses in using (P) in light of our critical examination of nutrient standards.

First, there are a number of problems in making a choice for the levels $r_j, j \epsilon J$. The very act of choosing fixed levels implies that any levels below those chosen are totally unsatisfactory. In view of the tentative nature of published standards and the ability of individuals to adapt to various levels of intake, this conclusion appears to be unwarranted. Beyond this problem, there are two practical implications in making this choice.

If the chosen standards cannot be satisfied, it is not clear how to proceed. Should the problematical levels be lowered? If they are lowered, what new levels should be chosen? Will nutrient inbalances result? These are all questions without clear answers.

The second practical implications in choosing a recommended or minimal set of standards $r_j, j \epsilon J$, is that by simply meeting the chosen levels, a diet of the maximum quality possible is not provided to the population of concern. Even if the chosen standards can be met, is adequate nutrition guaranteed? The answer to this question may be in the affirmative, but in view of the uncertainty of the recommended levels, why not seek to provide a diet with the maximum value possible? Even when nutritionists have a strong feeling that the chosen standards are adequate for meeting the nutritional needs of most population segments, is it necessary to stop there? Nutrition enhances the quality of life, and the true goal is to maximize that quality.

A second major shortcoming in model (P) is that it fails to provide a satisfactory structure for representing protein requirements. Our examination of nutrient recommendations showed that protein requirement levels cannot be defined unless the protein quality of the diet, on the average, is also known. While the protein quality of a

food or a set of foods may depend upon the level of intake or the age of the consumer, such hypotheses have yet to be absolutely proven. However, even under the assumption that the quality of the protein in a food or diet is independent of the level of intake or the age of the consumer, the model given by (P) is not adequate for representing protein requirements.

The protein quality of a food or diet is commonly measured by its net protein utilization (NPU). This is the ratio of the nitrogen actually used by the body to the total nitrogen consumed. The NPU of a diet is a fixed number between zero and one. The usual approach in specifying protein requirements is to increase the reference protein requirement r_2' for a population by a factor of $\left[1/NPU_x\right]$, where NPU_x represents the average NPU of the diet provided by plan x. The total protein requirement r_2 then becomes

$$r_2 = r_2'/NPU_x,$$

and the protein constraint in model (P), given by

$$f_2(x) \geqq r_2,$$

implies that

$$\left[NPU_x\right]\left[f_2(x)\right] \geqq r_2'. \qquad (1)$$

Inequality (1) states that the total protein $f_2(x)$ consumed under plan x, multiplied by the fraction of the total protein actually utilized, must meet or exceed the reference protein requirement. That is, the number of units of reference protein available physiologically should be at least r_2'.

The fallacy in the above procedure is that the value NPU_x cannot be specified a priori, i.e., it cannot be determined until the types of foods present in the diet and the quantities in which they are

present are known. Only the optimal food mixture given by the solution to model (P) can provide this information. Hence, procedures which employ the approach outlined above for specifying protein requirements in (P) may not be adequate.[8]

In addition to problems relating to the use and determination of values for the constants $r_j, j \in J$, and to the problem of prespecifying protein quality, model (P) fails to show the relationship among nutrients. For example, if the level of calories $f_1(x)$ in an optimal solution to (P) is marginal, this can increase the requirement for protein [48]. Since r_2, the protein requirement utilized in (P), is a fixed real number and does not vary with x, model (P) cannot take this into account.

Another potential weakness of model (P) is the fact that it employs a single objective function. A nutrition plan ought to maximize the supply of a number of nutrients. It is difficult to perturb the values r_j, one at a time, in order to do so. A more direct approach would be to employ a vector maximization model, which seeks to simultaneously "maximize" a number of nutrient levels $f_j(x), j \in J_S$, where $J_S \subseteq J$, subject to resource and nonnegativity constraints and, perhaps, to some nutritional constraints (See [4], [6], [29], [52]). An alternate approach might be to maximize some function of the nutrient levels $f_j(x), j \in J$.

These observations motivate the development of a nutrition planning model that overcomes the disadvantages of model (P). The bicriterion model that follows employs a dietary utility function and a structure for representing the protein content of a diet that alleviate the nutritional deficiences of model (P). By maximizing the

utility of the diet, our bicriterion model also allows for optimal nutrition planning.

3.2 The Bicriterion Model

Let the decision variables $x_k, k \in K$, the functions $f_j(x), j \in J$, and $g_i(x), i \in I$, and the constants $b_i, i \in I$ be defined as before. Assume that the indices $j \in J = \{1,2,\ldots,p\}$ representing nutrients have the meanings specified by Table 1, where $J_{VM} = \{11,12,\ldots,p\}$. Let $r_j, j \in J_{VM}$, represent annual recommendations for vitamin and mineral intakes.

KEY TO NUTRIENTS

INDEX	NUTRIENT
$j = 1$	Kilocalories
$j = 2$	Total Protein
$j = 3$	Isoleucine
$j = 4$	Leucine
$j = 5$	Lysine
$j = 6$	Methionine - Cystine
$j = 7$	Phenylalanine - Tyrosine
$j = 8$	Threonine
$j = 9$	Tryptophan
$j = 10$	Valine
$j \in J_{VM}$	Vitamins and Minerals

TABLE 1

Let u represent the number of units of reference protein provided annually. Although the essential amino acid composition of such a

protein is not precisely known, the most recent pattern recommended by the FAO will suffice for our purposes [48, p. 63]. For each $j \varepsilon J_E = \{3,4,...,10\}$, let k_j represent the number of units of essential amino acid j required per unit of reference protein. These values are obtained from the reference protein essential amino acid pattern. In addition, let k represent the annual caloric supply. Let $\mathbb{R}_+^2 = \{(k,u) \varepsilon \mathbb{R}^2 | k \geqq 0, u \geqq 0\}$.

Assume that a continuous dietary utility function h(k,u), defined on $\mathbb{R}_+^2$ and nondecreasing in each argument, is available which specifies the value of the annual national diet in terms of its caloric content k and reference protein content u.[9,10] Since calories and proteins are especially critical in food supplies, and protein-calorie malnutrition is the main malnutrition problem in the world, the most natural initial estimate for a measure of dietary value is in terms of these two nutrients.

The assumption that the utility function h(k,u) is nondecreasing in each argument implies that increased intakes of calories and reference protein are not detrimental. Although there is strong evidence that excessive caloric intakes, especially when fats represent more than one quarter of the total, can lead to heart disease [32, p. 46] [71], such levels will rarely, if ever, be attainable in developing nations. Recent evidence has shown that extremely high intakes of protein, on the order of 300 grams per day, may also be harmful in certain cases [8]. Again, the probability that such intakes will be attained for a given developing nation or population on an average per caput basis is miniscule. Thus, the assumption that h(k,u) is nondecreasing in each argument appears to be a sound one for nutrition planning in developing nations.

The bicriterion mathematical programming model (BPM) for nutrition planning in developing nations seeks to maximize the value of the diet subject to certain nutritional and resource constraints.[11] The model can be stated: Find x so as to

$$\max \quad h(k,u) \qquad (2)$$

subject to

$$f_1(x) = k \qquad (3)$$

$$f_2(x) - s = u \qquad (4)$$

$$f_j(x) \geqq k_j u \quad \forall\ j \in J_E \qquad (5)$$

$$s \geqq .67u \qquad (6)$$

$$u \leqq 4.0s \qquad (7) \qquad \text{(BPM)}$$

$$f_j(x) \geqq r_j \quad \forall\ j \in J_{VM} \qquad (8)$$

$$g_i(x) \leqq b_i \quad \forall\ i \in I \qquad (9)$$

$$x_k \geqq 0 \quad \forall\ k \in K \qquad (10)$$

$$k,s,u \geqq 0. \qquad (11)$$

Constraints (3), (4), and (5) define the annual supplies of calories k, reference protein, and total protein $f_2(x)$.[12] From (5), each unit of reference protein u requires at least k_j units of essential amino acid j for each $j \in J_E$. The supply of some essential amino acid $j' \in J_E$ will be the critical, or limiting, amino acid.[13] Then, for any optimal solution (x^*,k^*,s^*,u^*) the supply $f_{j'}(x^*)$ of this limiting amino acid equals $k_{j'}u^*$. For some other essential amino acid supplies $f_j(x^*)$, $j \neq j'$, constraint (5) will hold as a strict inequality at optimality. These excess amino acids, together with the excess nonessential amino acid supply in the protein, are accounted for by the variable s in (4). To see this, notice that, from (4), the total protein supply $f_2(x)$ for any feasible plan is given by

$$f_2(x) = u + s\ . \qquad (12)$$

Since u represents the essential and nonessential balances required in reference protein, s must represent the excess essential and nonessential amino acids in the total protein supply.

The constraints (6) and (7) specify that the NPU of the diet will be allowed to vary within certain limits. Since the NPU of a diet is the ratio of utilized protein to total protein ingested, the NPU of any feasible solution, NPU_x, from (12), is approximated by

$$NPU_x = u/(u + s).^{14} \qquad (13)$$

Because the NPU for national diets usually varies between 0.6 and 0.8, these values are used as bounds for NPU_x in (BPM).[15] Using (13), the restrictions given by

$$0.6 \leqq NPU_x \leqq 0.8$$

reduce to (6) and (7). It should be noted that when caloric supplies are marginal or inadequate, the approximation given by (13) is less adequate. In these cases, protein is at least partially directed towards providing energy.

Notice that in defining the caloric and protein supplies in (3) through (7), no minimal supply goals are required for calories, reference protein, total protein, or essential amino acids. Further, both the protein quality and quantity can vary.[16]

The constraints in (8) state that certain levels for vitamin and mineral supplies should be attained. In (9), the resource constraints of model (P) are repeated, and (10) and (11) are nonnegativity conditions.[17]

The nutrition planning model (BPM) does not require planners to choose fixed nutritional requirement levels r_j for calories, reference protein, total protein, or essential amino acids. As a result, there

is no critical point below which supplies for these nutrients are deemed inadequate. Furthermore, for these nutrients, the infeasibility problem of model (P) is no longer an issue. Since no standards are set for them, it is impossible for (BPM) to lack a feasible solution due to a failure in ability to satisfy caloric, protein, or essential amino acid levels. Furthermore, the objective function (2) in (BPM) allows for optimal nutrition planning. Regardless of whether various fixed requirement levels for calories, proteins, and essential amino acids can be satisfied or not, the best possible diet in terms of the utility measure specified by function h(k,u) will be provided.

In addition, the quality of protein is allowed to vary in (BPM), since the prespecification of NPU is not required. It will depend upon the feasible solution under consideration, as shown in (13). On the other hand, a value for the NPU of the recommended diet, although, in fact, unknown until a nutrition plan is presented, must be fixed in order to set protein requirements in model (P). This is a crucial factor, since in developing nations, it may be less costly to supply protein needs through large supplies of low quality protein, such as is found in grains and vegetables.[18]

Although the objective function h(k,u) may not necessarily imply any physiological relationships between calories and reference protein, it provides a measure for evaluating the impact of different mixes of these two nutrients. In model (P), however, no such measure is provided. Furthermore, the bicriterion nature of h(k,u) allows for the simultaneous consideration of caloric and reference protein supplies. This is more appropriate than considering a single factor in the objective function, such as cost or even the supply of some single nutrient. For the problem of nutrition planning is inherently multi-

dimensional, since the supplies of a number of nutrients must be considered. The bicriterion model (BPM) is a first step towards consideration of this multidimensionality.

An explanation of the rationale behind the use of reference protein u, rather than total protein, as the second argument in the dietary utility function is in order. In order to satisfy protein needs, nutritionists recommend that 60 to 70 per cent of the protein that is physiologically absorbed by the system be retained for actual use [32, p. 71] [37, pp. 688-690]. The figures for infants and children are even higher. Since NPU values for diets in developing nations may be as low as 0.6, and infants and children form a large portion of their populations, these diets may not be satisfying these recommendations. One way to help guarantee their satisfaction is to provide "good" quality protein. By maximizing the dietary value in terms of calories and reference protein, as opposed to total protein, model (BPM) provides a vehicle for attaining the minimal protein quality levels that nutritionists recommend.

3.3 The Dietary Utility Function

A key component in model (BPM) is the dietary utility function h(k,u). While a number of sources have suggested that such a function would be useful for large-scale nutrition planning, to date, it appears that no specific candidate for that function has been presented [7, p. 27] [17, pp. 188-189] [55, pp. 112-114]. Details of the development of the dietary utility function utilized in this study are given below.

In view of the sensitivity of one-through four-year-olds to caloric and reference protein intakes, mortality rates for this age group were compared to national caloric and reference protein supplies for various

nations at various times. In order to minimize the effects of health care, sanitation, and environmental conditions, the per cent of all deaths for all ages in a given period that occurred in the one-through-four age bracket was examined. Reference protein supplies were derived from total protein supplies by estimating the protein quality of the various national diets. These dietary and mortality data are presented in Table 2.

On the basis of these data, two potential candidates were examined as potential measures for the value of a diet in terms of its average daily per caput caloric and reference protein content. Both utilized the per cent mortality z due to one-through four-year-olds as a measure of value, the smaller values for z reflecting more valuable diets.

The first form, given by

$$h_1(k,u) = Ck^{-\gamma_1} u^{-\gamma_2}, \qquad (14)$$

where $C, \gamma_1, \gamma_2 > 0$, is defined on $\{(k,u) | k>0, u>0\}$.[19] The second form, given by

$$h_2(k,u) = AR_1^{(k-15)} R_2^{(u-20)}, \qquad (15)$$

where $A, R_1, R_2 > 0$ and $R_1, R_2 < 1$, is defined on $\{(k,u) | k>15, u>20\}$, since national diets which do not exceed 1500 calories and 20 grams of reference protein per caput per day do not exist in Table 2. Both functions are continuous, strictly decreasing in each of k and u, and are convex on their domains with diminishing marginal returns. As a result of the latter fact, the benefit of an increase in the supply of one or both nutrients is not as great when present levels of intake are high as it is when they are low. This property has been hypothesized for prospective dietary utility functions by Meadows, et al. [55, pp. 112-114].

TABLE 2. PER CENT CONTRIBUTION OF 1-THRU 4-YEAR-OLDS TO TOTAL MORTALITY VS. NATIONAL CALORIC AND REFERENCE PROTEIN SUPPLIES*

i	CALORIC SUPPLY k (100's of kcal/caput/day)	REFERENCE PROTEIN SUPPLY u (grams/caput/day)	% CONTRIBUTION z_i OF 1-THROUGH 4-YEAR-OLDS TO TOTAL MORTALITY	COUNTRY AND TIME PERIOD
1	32.33	70.9	.5	UNITED KINGDOM, 1964-66
2	31.90	67.8	.9	NETHERLANDS, 1964-66
3	29.64	69.1	.7	AUSTRIA, 1964-66
4	30.00	73.6	.6	FINLAND, 1965
5	28.27	65.0	1.3	ISRAEL, 1964-66
6	29.01	73.3	1.0	GREECE, 1964-66
7	28.10	57.4	3.6	ARGENTINA, 1962
8	27.69	54.0	7.7	TURKEY, 1964-66
9	25.80	70.5	2.8	URUGUAY, 1948
10	24.90	46.8	8.5	KOREA, 1969
11	25.00	47.5	7.8	CYPRUS, 1950
12	23.67	42.8	11.1	VENEZUELA, 1967
13	23.80	41.1	9.8	TAIWAN, 1966
14	27.30	46.2	7.8	S. AFRICA, 1966
15	24.50	38.4	10.2	PARAGUAY, 1962
16	21.60	39.0	13.8	VENEZUELA, 1950
17	25.41	41.5	8.8	BRAZIL†, 1960
18	22.34	34.0	12.2	COSTA RICA, 1965
19	21.20	30.0	15.8	COLOMBIA, 1969
20	21.00	28.2	15.9	VIET-NAM, 1964
21	19.20	31.2	20.6	PERU, 1948
22	19.80	28.2	21.1	ECUADOR, 1968
23	19.30	29.2	20.9	HONDURAS, 1964-66
24	19.52	29.0	23.0	GUATEMALA, 1965
25	17.65	27.5	26.8	BOLIVIA, 1964-66
26	18.77	28.8	19.0	EL SALVADOR, 1965
27	19.95	29.8	17.4	PAKISTAN, 1964-66
28	21.00	28.8	17.3	COLOMBIA, 1964
29	22.26	30.5	15.5	THAILAND, 1966
30	32.20	71.8	.5	DENMARK, 1964-66

† Guanabara State Only.

* Sources: [21], [25], [60], [78], [79], [84]

In order to test whether either of the forms given by (14) and (15) could provide an "acceptable fit" for the data in Table 2, a regression analysis was run for each. The multiple regression capability of SPSS (Statistical Package for the Social Sciences) [59], implemented on Northwestern University's CDC 6400 computer, was utilized for this purpose. The utility function $h_2(k,u)$ given by (15) was found to provide an acceptable fit for the 30 data points in Table 2 when $A = 56.6077216$, $R_1 = .9143561$, and $R_2 = .9489300$, at a significance level of .05.

Some caveats are in order. The implication should not be drawn that the dietary utility function $h_2(k,u)$ with these values for A, R_1, and R_2 is to be applied in any general nutritional situation. The function is not intended to imply any physiological relationships, and no inferences at the individual nutritional level should be drawn. Even at the national level, the functional values themselves have no absolute meaning and are intended only as relative measures of value. Furthermore, the function is not applicable to any nation at any given time. Such a universal national nutritional dietary function would probably depend upon a number of additional variables. These would include, for example, levels of vitamin and mineral supplies, the proportion of the total population represented by one-through four-year-olds, and the extent to which children within this age group receive their fair share of family food supplies.

4. APPLICATION TO NUTRITION PLANNING FOR COLOMBIA, SOUTH AMERICA

The bicriterion mathematical programming model for nutrition planning (BPM) presented in §3 is suitable for nutrition planning for developing nations or for selected population groups or geographical

regions within these nations. Let us analyze the results of its application to Colombia, South America. We begin with a brief overview of the agricultural and nutritional setting in Colombia.

4.1 The Agricultural and Nutritional Situation in Colombia

Colombia is a nation with abundant and rich natural resources and large variations in climate. As a result, a wide variety of agricultural activities takes place in Colombia. Major domestic crops include corn, wheat, rice and plantains, with a number of fruits, vegetables, pulses, nuts, and oilseeds grown in lesser amounts. The main export crop is coffee, which accounts for two-thirds of Colombia's export earnings [46, p. ii]. Bananas, cotton, sugar, and tobacco are also exported in significant amounts. The livestock industry, involving mostly beef cattle and dairy production, is also a major component of the agricultural sector. Despite 1170 miles of ocean front and considerable freshwater fishery potential, however, fisheries account for less than one per cent of the gross domestic product [83, p. 297].

The mainstay of most Colombian diets is corn. Long-grained rice is also a principal dietary item in many areas, and plantains are an important feature in most diets. In certain areas, dry beans are a dietary staple, and cassava has been a traditional food for many years, especially in rural areas.

Although a wide variety of fruits and vegetables is grown, none is an important dietary component due to poor storage and transport facilities. Beef is less expensive than poultry, lamb, and mutton, so that most of the animal produce consumed is beef. Almost no fish is consumed in inland areas.

Despite the excellent conditions for agriculture and the variety of crops grown in Colombia, malnutrition is a serious problem for a significant portion of the populace. A number of factors can be cited which contribute to this problem.

Agricultural production suffers from low yields, especially for domestic crops. Traditional agricultural inputs are the norm, and most farms are small and poorly managed. Agricultural agencies, technologies, and marketing are geared mainly towards export crops. As a result, both corn and wheat have been imported in large quantities in recent years.

Storage and transport facilities are poor or entirely absent in many areas, so that few people consume fruits, vegetables, pulses, or fish. Processing methods are poor, with no established standards of quality existing for many products. Wastage of crops during harvesting, storage, and transport abounds, with up to 25 per cent of the crop being lost for some fruits in this manner [21].

In addition, the population in Colombia has been growing at annual rates of 2.8 to 3.2 per cent in recent years [80, p. 78] [83, p. 41]. Such increases represent an important factor in Colombia's nutritional situation. Poverty is also a key factor, and many families are financially limited in their ability to purchase adequate diets. Of the total annual income in Colombia, about 57 per cent is concentrated in the hands of 20 per cent of the people [81, p. 355].

Approximately one-half of the population is functionally illiterate, and in 1964 only 14 per cent of Colombians had more than four years of education [3, p. 13] [80, p. 12]. As a result, significant portions of the population lack a knowledge of their basic nutritional requirements. Furthermore, health services are too expensive or inaccessible for many, so that infections and diseases go unchecked in

many areas [1]. This increases nutritional requirements and aggravates the supply problem. Losses of nutrients in home cooking and consumption can also be significant, especially among lower income groups.

All of these problems have led to serious malnutrition in certain population groups in Colombia. Conclusions of various food and nutrition surveys can be summarized as follows (see [18], [39], [40], [42-45], [47], [80]):

1. Protein-calories malnutrition (PCM) is a serious problem in Colombia, especially among children under six years of age. Perhaps as many as 56 per cent of the children in this group have PCM to some degree [80, p. 21].

2. Chronic subnutrition in adults is the norm, in many areas of Colombia, rather than the exception. More than one-half the adult population probably lacks adequate intakes of at least one nutrient [80, p. 50].

3. Malnutrition is more common among pregnant and lactating mothers than in the general population.

4. In addition to calories and proteins, large portions of the population have inadequate intakes of vitamin A, iron, calcium, and riboflavin. Vitamin A and iron deficiency are especially prevalent. Intakes of thiamin and niacin may also be marginal in certain areas.

Least cost diet studies [20] [41] confirm that supplies of calories, proteins, vitamin A, iron, calcium, and riboflavin are not adequate, while a nutrition planning model [74] indicates that insufficient income is one important factor in malnutrition in Colombia.

4.2 Highlights of the Colombia Model

A bicriterion mathematical programming model for nutrition planning in Colombia, based upon model (BPM), was formulated and solved in an attempt to assess the usefulness, feasibility, and tractability of using such techniques in real-world situations. The model was formulated for the year 1972, and only agricultural and nutritional technologies and methods practiced that year are considered. In order to plan for Colombia's future needs, a similar model could be formulated utilizing projected population levels, crop yields, and other estimates of the pertinent data for the year in question.

In the Colombian model, the population is divided into four income groups. Two sets of nutritional constraints are considered. The first set requires that each of the upper two income groups must purchase diets with nutrient levels at least as great as actual 1972 levels. The rationale is that these groups would not be willing to change their consumption patterns for the benefit of the poorer groups. The second set of nutrient constraints define the caloric, reference protein, total protein, and essential amino acid consumption for the two lower income groups, taken together, based upon model (BPM). In addition, lower bounds on the consumption of seven other nutrients are set for these income groups as shown in model (BPM). These levels are based upon recommendations of the FAO [26], [50], [51]. The seven nutrients considered are calcium, iron, vitamin A, thiamin, riboflavin, niacin, and vitamin C. The objective is to maximize the value of the diet $h(k,u)$ of the two lower income groups, based upon the average daily per caput consumptions of calories k and reference protein u within these groups.

Agricultural products can be produced internally or imported in the model. Exports are also allowed. Only those agricultural products actually produced, imported, or exported at some time within the recent past in significant amounts are considered in defining each of these activities. This results in 65 production alternatives and an allowance for 26 import-export activities. For each production activity, one or more agricultural commodities is produced, so that, in all, 70 agricultural commodities are considered. Certain activities, such as fish production, are fixed at 1972 levels due to inadequate data on their yields or other factors. Each income group can purchase from among the available food commodities, subject to its economic ability to afford to do so.

A number of economic, agricultural, and trade policies and goals are incorporated into the constraints. The impetus for doing so is to provide a nutrition plan that conforms with current agricultural and economic policies in Colombia. (See [3], [46], [81], and [83] for a discussion of national and international economic problems and goals in Colombia.)

One set of constraints provides for the generation of agricultural employment under the nutrition plan to be provided. Another set requires that a certain minimal level of income redistribution take place under the plan, both in the agricultural sector and within the general populace. Rural unemployment and the concentration of wealth within a small sector of the population are two important problems in Colombia's domestic economy.

In addition to constraints relevant to the internal economic situation, the model constrains agricultural imports and exports in a manner consistent with international trade policies stated by the Colombian

government. Agricultural exports are promoted and imports kept within certain bounds as desired by government programs. For example, exports of coffee, bananas, and sugar are bounded below by their actual 1972 levels (or some major fraction thereof), while imports of corn and wheat are restricted to lie at or below actual 1972 levels. Furthermore, since agricultural export earnings are a major contribution to Colombia's total earnings from international trade, the net earnings from agricultural trade of the commodities considered are restricted to be at least 95 per cent of their actual 1972 level.

Other important constraints in the model are on total land availability, on land availability by soil types, on availability of feeds, fertilizers, and pesticides, and on production costs. The costs of production are a particularly important factor in Colombia, where farmers have great difficulty obtaining credit.

A number of factors that can significantly affect the nutrient supply for human consumption are given explicit consideration in the constraints. For example, allowances are made for losses of nutrients in cooking, for wastage of foods in processing, storage, and transport, for the contribution of breast feeding to the total nutrient supply, for industrial, feed, and seed uses of crop and livestock produce, and for the fact that certain portions of foods are inedible.

To restrict excessive supplies of certain foods and thereby achieve a certain level of palatability for the diets, upper bounds are placed on each agricultural activity level. Because marketing facilities prohibit large expansions in the production of fruits and

vegetables, most of these activities are either fixed at actual 1972 levels or are restricted to increases of at most 20 per cent over their 1972 levels. In addition, a fixed portion of cropland is set aside as idle land under the plan.[20]

4.3 Optimal Nutrition Planning for Colombia

The bicriterion nutritional planning model for Colombia utilizes 413 decision variables and 285 linear equations and inequalities. The objective function involves the maximization of a bicriterion concave function, strictly increasing in each argument. The algorithm developed in [4] was used to obtain the optimal solution. The parameterization required to implement the solution technique was accomplished with the aid of the Apex II [15], [16] parametric linear programming option on Northwestern University's CDC 6400 computer. Intermediate solutions were recorded at regular intervals, and the total CPU time required to obtain the optimal solution using this procedure was approximately ten minutes.

The optimal nutrition plan supplies k = 2569.6 calories and u = 35.829 grams of reference protein per person per day for individuals in the lower income groups. The NPU of their diet is 0.8 under this plan, with a total protein consumption of 44.787 grams per person per day. Such intakes satisfy the energy, reference protein, and total protein requirements for these two groups based upon the latest FAO standards [48]. In fact, this optimal diet exceeds FAO recommended energy intakes by 23.88% and recommended reference protein intakes by 6.56%. The limiting amino acid is lysine, followed by threonine. This is consistent with our expectations, since these two essential amino acids are scarce in many natural foods. Only

iron and vitamin A are binding among the seven nutrient constraints for these two income groups. The consumption of all nutrients considered shows a marked improvement over present levels for these groups.[21]

The solution technique yields a corresponding optimal linear program whose dual variables indicate the relative scarcity of nutrients under the optimal plan. From these values, it is evident that of the two nutrients whose supplies are binding for the lower income groups, vitamin A is more critical than iron.

In addition, constraints on energy, protein, vitamin A, thiamin, and riboflavin are binding for both upper income groups at their 1972 levels of consumption, and iron is binding for one of these groups. These results are consistent with earlier estimates of critical nutrients in Colombia's nutrient supply. They also indicate that a reduction in consumption by the upper income groups of any of these nutrients could benefit the value of the diet of the lower two groups.

The major shifts in food consumption, production, and trade activities that allow these improvements in nutrient consumption to take place for the lower income groups can be summarized as follows. Marked increases in consumption of barley, arracacha, yams, peanuts, palm oil, carrots, and pig offals are recommended for the lower income groups. In addition, significant consumption of wheat flour, white and yellow corn, corn meal, plantains, centrifugal sugar, chicken, and whole milk are called for. Carrots yield an excellent source of vitamin A for these groups, yams, arracachia, and plantains an inexpensive energy source, and whole milk is recommended for its overall nutritional value. Even animal products such as beef, chicken, and pig offals are called for in the diet of the lower income groups

due to their excellent protein quality.

Among items not appearing in any of the four diets or appearing at significantly reduced levels are rice, potatoes, chocolate, panela (brown sugar), and beans. These products are generally too expensive or not adequately nutritious to merit their appearance at levels that were current in 1972.

At the production level, certain established crops are not grown and others are grown at levels consistent with the consumption levels recommended. Among crops of potential value to lower income groups, given more land to produce them on, are carrots, cabbage, wheat, yams, plantains, peanuts, African palm, citrus fruits, and guava. Carrots are especially beneficial as a source of vitamin A and cabbage as a calcium source.

Remarkably, the composition of agricultural imports is virtually identical to that which actually occurred in 1972. On the export side, the major change is a recommendation that beer exports constitute approximately 25 per cent, by value, of the major agricultural exports considered. In 1972, no beer was actually exported at all.

Finally, agricultural resources binding in the optimal solution include protein feeds for livestock, pesticide supplies, agricultural capital, and total land. In view of the protein shortages in Colombia, it is not surprising that protein feed for livestock is not in excess supply in the plan.

In summary, the two lower income groups can not only achieve recommended intakes for energy and reference protein, but they can exceed these levels under the plan presented. In so doing, the value of their diet is maximized. Furthermore, the NPU of their resulting

diet is 0.8, a level heretofore assumed unattainable within developing nations. The upper income groups need not sacrifice the quality of their diets to help achieve these intakes for the lower groups. What is required are some shifts in consumption patterns, accompanied by increased production of certain grains, starches, vegetables, fruits, and animal products. Certain other products would be curtailed or discontinued. Agricultural trade would not be significantly modified, except for a sharp rise in the export of beer. The authors feel that the effort required to implement these changes would clearly be justified by the benefits obtained.

As a by-product of the solution technique, a number of <u>efficient</u> nutrient plans is generated.[22] By efficient we mean that given the outcome (k^1, u^1) of daily per caput calorie and reference protein intakes under any such plan, there can exist no other feasible solution to our nutrition planning model which supplies k' calories and u' grams of reference protein per caput per day to the two lower income groups such that $k' \geqq k^1$ and $u' \geqq u^1$ with strict inequality holding in at least one case. In view of the uncertainties in methods for assessing values of various levels of nutrient intakes, nutrition planners may prefer to utilize one or more of these alternative plans in preference to the one examined here. Some of the outcomes of the efficient nutrition plans generated during the search for the optimal plan are presented in Table 3. Given any well-defined dietary utility function $h'(k,u)$, nondecreasing in each argument, one of the efficient nutrient plans generated during the course of the solution procedure would solve the problem of maximizing $h'(k,u)$ subject to the constraints outlined here.

TABLE 3. OUTCOMES OF SOME

EFFICIENT NUTRIENT PLANS

k (100's calories/caput/day)	u (g ref. protein/caput/day)
18.84	44.13
19.10	44.05
19.20	44.01
19.26	43.98
19.38	43.88
19.60	43.69
19.83	43.49
20.19	43.18
20.32	43.07
20.52	42.89
20.76	42.60
20.90	42.40
21.27	41.91
21.34	41.81
22.32	40.46
22.78	39.83
23.34	38.97
25.58	35.99
*25.696	35.829
25.72	35.66
25.81	34.39
25.86	33.28
25.88	33.00
25.889	32.69
25.89	32.66

*This is the optimal outcome under the dietary utility function $h(k,u)$ employed in this paper.

5. SUMMARY AND CONCLUSIONS

Our examination of nutrient standards reveals that uncertainties exist as to proper methods for evaluating nutrient requirements, and that results so far are tentative. The implications of these conclusions for traditional mathematical programming approaches to nutrition planning for developing nations lead us to seek a new approach. The approach developed utilizes a bicriterion nutrition planning model that maximizes the value of the diet in terms of caloric and reference protein intakes. By using the bicriterion model, reliance upon fixed nutrient standards is not strong, and, furthermore, both the quality and quantity of the protein intake are optimized. The application of the model to the lower income groups in Colombia, South America reveals that these groups could achieve a diet that exceeds current recommended levels in energy and reference protein content. Furthermore, this diet contains high-quality protein and is not achieved at the expense of reduced intakes for other income groups in Colombia. Agricultural, economic, and trade activities under the plan recommended for Colombia appear to be consistent with government policies and goals. Our solution technique allows for the examination of resources critical for nutrition planning. In addition, a number of alternate efficient nutrition plans is generated as a by-product of this solution procedure. While the application is illustrative in nature, the results indicate that the approach is both feasible and tractable and that the benefits are considerable enough to warrant the serious consideration of the method by nutrition planners.

NOTES

1 The references cited are typical examples and are not intended to be exhaustive.

2 Many governments have formulated nutritional standards, but only selected references are cited herein.

3 A kilocalorie or Calorie is the amount of heat required to raise the temperature of one liter of water from 15 degrees centigrade to 16 degrees centigrade. A kilocalorie is equal to 1000 calories. However, for convenience, any reference to calories in the text will refer to kilocalories, since the smaller unit is never utilized in nutritional contexts.

4 The eight essential amino acids are isoleucine, leucine, lysine, methionine, phenylalanine, threonine, tryptophan, and valine. Histidine may also be an essential amino acid, but the evidence for this is incomplete. In addition, cystine is often listed in conjunction with methionine and tyrosine with phenylalanine, since cystine and tyrosine have a sparing effect on methionine and phenylalanine, respectively.

5 It should be born in mind that such nitrogen is a vital component of dietary protein. The term nonessential refers to the nonessential amino acid source of the nitrogen and does not mean to imply that this nitrogen is not needed for adequate human nutrition.

6 For a thorough discussion of energy and protein in nutrition, see Guthrie [32], Guyton [33], or Williams [82].

7 The model (P) may, in general, also involve additional constraints, represented either by equalities or inequalities. The levels representing annual nutrient requirements may apply to either an entire nation or to population groups within the nation.

8 Smith [68] has an alternate model to model (P) which overcomes this problem.

9 The interested reader is referred to [31] for an overview of utility theory.

10 In practice, it may be more convenient to measure dietary utility in terms of average daily per caput intakes of calories and reference protein, rather than in terms of annual supplies. We use average daily per caput intakes. In order to avoid unnecessary algebraic transformations, however, it is sufficient to examine annual supplies for illustrative purposes.

11 The diet may be on a national level or may pertain to some population group within the nation.

12 Notice that (3) can be modified to require that annual caloric supplies meet <u>or exceed</u> k, since, in this case as well, for any optimal solution (x^*, k^*, s^*, u^*), caloric supplies will equal k^*.

13 Often, either lysine, threonine, tryptophan, or methionine-cystine is the limiting essential amino acid in national nutrient supplies.

14 Since the protein quality of a diet also depends upon the levels of total protein, caloric, and specific amino acid intakes [64], the relationship implied by (13) is only an approximate one. The only assumption is that energy supplies are adequate.

15 The NPU of diets in developing nations often varies between 0.6 and 0.7, while in developed nations the values are usually near 0.8 [48, p. 73].

16 For a related approach that also allows protein quality to vary, see Smith [68].

17 Additional resource constraints, represented by inequalities or equalities, also may be present.

18 The proper mix of these foods, of course, can also provide protein with quality comparable to protein from more expensive sources.

19 This is a Cobb-Douglas function.

20 See [5] for a complete presentation of the Colombian model.

21 See [5] for a detailed analysis of nutrition, dietary intakes, and agricultural programs under the optimal plan.

22 Many sources (For example, [4], [6], [29], [30], [52]) examine the concept of <u>efficient solutions</u> for vector maximization problems.

REFERENCES

1. BADGLEY, R. F., "Social Science and Health Planning: Culture, Disease and Health Services in Colombia," Milbank Memor. Fund Quarterly 46, No. 2, 1-352 (1968).

2. BALINTFY, J. L. and BLACKBURN, C. R., "A Significant Advance in Hospital Menu Planning by Computer," Institutions Magazine 54-56; 102-105 (1964).

3. BENNETT, G. A., Agricultural Production and Trade of Colombia, United States Department of Agriculture, Economic Research Service Report No. 343, Washington, D. C., 1973.

4. BENSON, H.P., "An Approach to Bicriterion Mathematical Programming," Paper Presented at ORSA/TIMS National Meeting, Las Vegas, November, 1975.

5. ____________, "A Bicriterion Mathematical Programming Approach for Nutrition Planning in Developing Nations," Ph.D. dissertation, Northwestern University, Evanston, Illinois, 1976.

6. ____________, and MORIN, T. L., "The Vector Maximization Problem: Proper Efficiency and Stability," SIAM Journal on Applied Mathematics (to appear).

7. BERG, A., The Nutrition Factor: Its Role in National Development, The Brookings Institute, Washington, D. C., 1973.

8. CALLOWAY, D. H., "Dietary Components that Yield Energy," Env. Biol. and Med. 1, 175-186 (1971).

9. CANADIAN BULLETIN OF NUTRITION, "Recommended Daily Intakes of Nutrients Adequate for the Maintenance of Health Among the Majority of Canadians," 6, No. 1 (1964).

10. CHAPIN, R., et al., "L. P. of Space Rations for Acceptability and Nutritional Adequacy," Federation Proceedings 26, 304 (1967).

11. CHAPPELL, A. E., "L. P. Cuts Costs in Production of Animal Feeds," Opns. Res. Qrtly, 25, 19-26 (1974).

12. CHEN, J. T., "Quadratic Programming for Least-Cost Feed Formulations Under Probabilistic Protein Constraints," Amer. J. Agric. Econ. 55, 175-183 (1973).

13. COLE, H. S. D., et al. (eds.), Models of Doom, Universe, New York, 1973.

14. COLOMBIA, Division de Investigacion, Recomendacion Diaria de Calorias Y Nutrientas para la Poblacion Colombiana, Publicacion EPI-65-03, Ministerio de Salud Publica, Bogotà, 1965.

15. CONTROL DATA CORPORATION, APEX II Reference Manual, Control Data Corporation, Minneapolis, 1973.

16. ______________________, APEX II User's Guide, Control Data Corporation, Minneapolis, 1974.

17. CORREA, H., "Nutrition, Working Capacity, Productivity, and Economic Growth," pp. 188-191 in Proceedings: Western Hemisphere Nutrition Congress II, 1968, American Medical Association, Chicago, 1969.

18. DEPARTAMENTO ADMINISTRATIVO NACIONAL DE ESTADISTICA (DANE), Estructura del Consumo Relacionado con la Salud, Bogotà, 1971.

19. ECKSTEIN, E. F., "Menu Planning by Computer: The Random Approach to Planning for Consumer Acceptability and Nutritional Needs," Ph.D. dissertation, Kansas State University, Manhattan, 1969.

20. FLORENCIO, C. A., and SMITH, V. E., "Efficiency of Food Purchasing Among Working-Class Families in Colombia," J. Amer. Diet. Assoc. 55, 239-245 (1969).

21. FOOD AND AGRICULTURE ORGANIZATION OF THE UNITED NATIONS, Food Balance Sheets: 1964-1966,FAO, Rome, 1967.

22. ______________, Report on Calorie Requirements, FAO Nutritional Studies, No. 5, FAO, Rome, 1950.

23. ______________, Report on Calorie Requirements, FAO Nutritional Studies, No. 15, FAO, Rome, 1957.

24. ______________, Report on Protein Requirements, FAO Nutritional Studies, No. 16, FAO, Rome, 1957.

25. ______________, Second World Food Survey, FAO, Rome, 1952.

26. FAO/WHO EXPERT GROUP, Calcium Requirements, FAO Nutrition Meetings Report Series, No. 30, FAO, Rome, 1962.

27. FOOD AND NUTRITION BOARD, Recommended Dietary Allowances, 7th Revised Edition, National Academy of Science Publication No. 1694, Washington D. C., 1968.

28. ______________, Recommended Dietary Allowances, 8th Revised Edition, National Academy of Sciences, Washington D. C., 1974.

29. GEOFFRION, A. M., "Proper Efficiency and the Theory of Vector Maximization," J. Math. Anal. and Appl. **22**, 618-630 (1968).

30. ______________, "Solving Bicriterion Mathematical Programs," Opns. Res. **15**, 39-54 (1967).

31. GORMAN, W. M., "The Structure of Utility Functions," Rev. Econ. Stud. **35**, 367-390 (1968).

32. GUTHRIE, H. A., Introductory Nutrition, Mosby, St. Louis, 1971.

33. GUYTON, A. C., Textbook of Medical Physiology, Saunders, Philadelphia, 1966.

34. HARPER, A. E., "Recommended Dietary Allowances: Are They What We Think They Are?," J. Amer. Diet. Assoc. **64**, 150-156 (1974).

35. HEADY, E. O., Agricultural and Water Policies and the Environment, The Iowa State University Press, Ames, Iowa, 1972.

36. HEADY, E. O. (ed.), Economic Models and Quantitative Methods for Decisions and Planning in Agriculture, The Iowa State University Press, Ames, Iowa, 1971.

37. HEGSTED, D. M., "Amino Acid Fortification and the Protein Problem," Am. J. Clin. Nut. 21, 688-692 (1968).

38. ______________, "Energy Needs and Energy Utilization," Nut. Rev. 32, 33-37 (1974).

39. INSTITUTO COLOMBIANO DE BIENESTAR FAMILIAR, Información Sobre Nutrición y Alimentación para el Plan Nacional de Salud, Dirreción de Nutrición, Bogotà, 1967.

40. ____, Informe Sobre Siete Años de Programas de Nutrición en Colombia, Dirrección de Nutrición, Bogotà, 1970.

41. INSTITUTO DE INVESTIGACIONES TECNOLOGICAS, Metodología para el Analisis Multidisciplinary de Nutrition, 3 Vols., Bogotà, 1972.

42. ____, "Metodologías para un Analisis Multidisciplinario de la Desnutrician," Tecnologia, No. 79, 3-19 (1972).

43. INSTITUTO NATIONAL DE NUTRICION, Metas de Disponibilidad y Consumo de Alimentos para Colombia--1965-1974, Bogotà, 1964.

44. ____________________________, Resumen de las Encuestas Alimentarias Realizadas par INN de 1963 à 1965, Bogotà, 1968.

45. ____________________________, Resumen de las Encuestas Nutricionales Realizadas Entre 1963 y 1968, Bogotà, 1970.

46. INTERAMERICAN BANK OF RECONSTRUCTION AND DEVELOPMENT, The Coffee Economy of Colombia, IBRD, New York, 1968.

47. INTERDEPARTMENTAL COMMITTEE ON NUTRITION FOR NATIONAL DEFENSE, Nutrition Survey, May-August, 1960: Colombia, Washington D. C., 1961.

48. JOINT FAO/WHO EXPERT COMMITTEE, Energy and Protein Requirements, FAO Nutrition Meetings Report Series, No. 52, FAO, Rome, 1973.

49. JOINT FAO/WHO EXPERT GROUP, Protein Requirements, FAO Nutrition Meetings Report Series, No. 37, FAO, Rome, 1965.

50. ________________________, Requirements of Ascorbic Acid, Vitamin D, Vitamin B_{12}, Folate, and Iron, FAO Nutrition Meetings Report Series, No. 47, FAO, Rome, 1970.

51. ________________________, Requirements of Vitamin A, Thiamine, Riboflavin, and Niacin, FAO Nutrition Meetings Report Series, No. 41, FAO, Rome, 1967.

52. KUHN, H. W., and TUCKER, A. W., "Nonlinear Programming," pp. 481-492 in Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, J. Neyman (ed.), University of California Press, Berkeley, California, 1950.

53. LEAGUE OF NATIONS, The Problem of Nutrition, Technical Commission of the Health Committee, Geneva, 1936.

54. McFARLANE, I. G., and VON HOLT, C., "Metabolism of Amino Acids in Protein-Calorie-Deficient Rats," Biochemical J. 111, 557-563 (1969).

55. MEADOWS, D. H., et al., The Limits to Growth, New American Library, New York, 1972.

56. MESAROVIC, M. and PESTEL, E., Mankind at the Turning Point, Dutton, New York, 1974.

57. MINISTRY OF HEALTH AND WELFARE, Nutrition in Japan, 1964, Tokyo, 1965.

58. NEWMAN, P., "Some Calculations of Least Cost Diets Using the Simplex Method," Bulletin of the Oxford University Institute of Statistics 17, 303-320 (1955).

59. NIE, N. H., et al., Statistical Package for the Social Sciences, McGraw Hill, New York, 1970.

60. ORGANIZATION OF AMERICAN STATES, America en Cifras (Various Yrs.)

61. PANNE, C., and POPP, W., "Minimum-Cost Cattle Feed Under Probabilistic Protein Constraints," Man. Sci. 9, 405-421 (1963).

62. PATWARDHAN, V. N., "Dietary Allowances for India," Indian Council on Medical Research, Special Report Series, No. 35, New Dehli, 1960.

63. SAI, F. T., "Food Nutrition and Family Health," Food and Population, Report No. 19, 28-30 (1974).

64. SAID, A. K., and HEGSTED, D. M., "Response of Adult Rats to Low Dietary Levels of Essential Amino Acids," J. Nutrition 100, 1363-1374 (1970).

65. SCOTT, J. T., JR., and BAKER, C. B., "A Practical Way to Select an Optimum Farm Plan under Risk," Am. J. Agric. Econ. 54, 657-660 (1972, Part 1).

66. SCRIMSHAW N. S., et al., "Protein Requirements of Man: Variations in Obligatory Urinary and Fecal Nitrogen Losses in Young Men," J. Nutrition 102, 1595-1603 (1972).

67. SMITH, V. E., "Agricultural Planning and Nutrient Availability," Nut. Rev. 28, 143-150 (1970).

68. _______________, "A Diet Model with Protein Quality Variable," Man. Sci. 20, 971-980 (1974).

69. _______________, "Linear Programming Models for the Determination of Palatable Human Diets," J. Farm Econ. 41, 272-283 (1959).

70. ______________, and YAGHOOBI-RAHMATABADI, H., "Agricultural Improvements and the Income Potential of Nigerian Agriculture," Econometric Workshop Paper No. 7.02, Michigan State University 1972.

71. STAMLER, J., et al., "Coronary Risk Factors," Med. Clin. N. Amer. 50, 229-254 (1966).

72. STICKNEY, R., et al., "Systems Approach to Nutrition Planning: Preliminary Considerations," Pittsburgh Instrument Society of America, 1973.

73. STIGLER, G. J., "The Cost of Subsistence," J. Farm Econ. 27, 303-314 (1945).

74. SUTTOR, R. E., "Adjustments in Agricultural Production to Attain Minimum Nutritional Requirements Consistent with Consumer Incomes," Agency for International Development, Bureau for Latin America, Sector Analysis Division, Analytical Working Document No. 16, February, 1974.

75. ______________, "Linear Programming Analysis of Nutrient Production Capacity of the Colombian Agriculture Sector," Agency for International Development, Bureau for Latin America, Sector Analysis Division, Analytical Working Document No. 11, April, 1973.

76. SWANSON, E. R., "Integrating Crop and Livestock Activities in Farm Management Activity Analysis," J. Farm Econ. 37, 1249-1265 (1955).

77. THOMAS, W., et al., "Separable Programming for Considering Risk in Farm Planning," Am. J. Agric. Econ. 54, 260-266 (1972, Part I).

78. UNITED NATIONS, Demographic Yearbook, Department of Economic and Social Affairs, U. N., New York (Various Yrs.)

79. ______________, Statistical Yearbook, Department of Economic and Social Affairs, U. N., New York (Various Yrs.)

80. UNITED STATES AGENCY FOR INTERNATIONAL DEVELOPMENT-COLOMBIA, Colombian Health Sector Analysis, May, 1974.

81. UNITED STATES DEPARTMENT OF DEFENSE, Area Handbook-Colombia, U. S. Govt. Printing Office, Washington, D. C., 1970.

82. WILLIAMS, S. R., Nutrition and Diet Therapy, Mosby, St. Louis, 1969.

83. WORLD BANK, Economic Growth of Colombia: Problems and Prospects, John Hopkins University Press, Baltimore, 1972.

84. WORLD HEALTH ORGANIZATION OF THE UNITED NATIONS, World Health Statistics Annual, WHO, Geneva (Various Yrs.)

85. WHO EXPERT COMMITTEE, Nutrition in Pregnancy and Lactation, WHO Technical Report Series, No. 302, WHO, Geneva, 1965.

86. ______________, Other Trace Elements Essential for Human Nutrition, WHO Technical Report Series, No. 532, WHO, Geneva, 1973.

87. YARON, D., "Applications of Math Programming to National Planning in Agriculture," Farm Econ. 10, 256-264 (1963).